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A photograph on a smaller scale was also obtained with the 4-inch Dallmeyer rapid rectilinear lens of 34 inches focus on July 26, exposure 125^{m}. The most successful photographs with the reflector are the one taken with an exposure of 80^{m} on July 24 and the one taken on August 1, which is reproduced in Plate 1. The photograph taken on August 1 shows a sharply defined head with nine distinct tails, also sharply defined, the longest extending to the edge of the field, a distance of 125′.

On the Semi-diameter, Parallactic, Inequality, and Variation of the Moon from Greenwich Meridian Observations, 1847·0 *to* 1901·5. By P. H. Cowell.

In the following paper an account is given of an investigation of the solar parallax from observations of the Moon.

The material used consists of all Greenwich meridian observations with the transit circle from 1847·0 to 1901·5. This interval consists of forty-eight periods of 400 lunar days. Each period of 400 lunar days is treated separately, and 400 lunar days are approximately equal to fifteen anomalistic months and fourteen lunations. Associated, therefore, with any particular value of D, the age of the Moon, there will be found in one period fourteen values of g, the mean anomaly, uniformly distributed round 360°, so that certain classes of error should be to a great extent eliminated in each period.

The tabular places used are based upon Hansen's tables with Newcomb's corrections, and Hansen's error of sign corrected. From 1847–1861 the tabular places are taken from vol. l. of the *Monthly Notices*, and Newcomb's corrections have been applied. From 1862 onwards the tabular places have been taken from the Greenwich annual volumes, and from 1862–1882 Newcomb's corrections and a correction for Hansen's error of sign have been applied.

After this all tabular places received the following corrections:

$$
\begin{aligned}
&+1''{\cdot}562 \sin D \\
&-0{\cdot}893 \sin (g+2\varpi-2J) \\
&-0{\cdot}676 \sin (g+2\varpi+3V-5E) \\
&-0{\cdot}548 \sin g \\
&+\{N+\Delta g+4''{\cdot}06 \sin \Omega\}\, 2e \cos g
\end{aligned}
$$

where in the last term N denotes Newcomb's corrections, and is therefore to be taken as zero from 1883 onwards; Δg is a correction to the mean anomaly investigated by myself; its value is $\Delta g = +4''{\cdot}63 - 13''{\cdot}99T + 4''{\cdot}743T^2$ (see *Monthly Notices,*

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1903 November) where the rest of the notation and the source of the corrections can be found. It is not necessary to repeat them here.

The present determination is therefore based upon 5647 observations, each observation being compared with a corrected theory, and therefore subject to a less accidental error; for a periodic error in the tabular places is of the nature of an accidental error when terms of a different period are under discussion (unless the argument of the error and the argument under discussion differ by a low multiple of D, the mean elongation). I believe, therefore, that the present discussion is based upon material of greater weight than any previously employed in deducing the solar parallax from lunar observations.

Twice the circumference of the circle, or 720°, has been divided into fifty-seven equal parts; and the observations grouped under the corresponding values of D. The fifty-seventh part of 720° is nearly equal to the movement of D in one lunar day. The difference between the true value of D and the value under which the observation is grouped is not necessarily less than half of 360°÷57; for in order to make the grouping mechanical the observations are not always grouped under the nearest value of D in the series of 57. The error may, in fact, amount to 7°, and its average value will be about 2°·5.

The periods of 400 lunar days are numbered from 86 to 133 because period No. 1 begins, with Airy's reductions, on 1750 September 13.

For the sake of greater clearness of explanations I give here the figures for period 86.

Table I.

Period 86.

Ref. No.	Value of D.	Sum of Errors in units of one-tenth of second of arc.	No. of observations.	Ref. No.	Value of D.	Sum of Errors in units of one-tenth of second of arc.	No. of observations.
0	0°·0			12	151°·6	−45	5
1	12·6			13	164·2	+ 4	4
2	25·3			14	176·8	− 5	3
3	37·9			15	189·5	+67	3
4	50·5	−12	1	16	202·1	+56	4
5	63·2			17	214·7	+42	1
6	75·8	+11	2	18	227·4	+ 7	2
7	88·4	+ 8	2	19	240·0	−10	1
8	101·0	−30	2	20	252·6	+61	3
9	113·7	+43	6	21	265·3	− 2	5
10	126·3	− 7	6	22	277·9	+35	2
11	139·0	+12	3	23	290·5	−14	4

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Ref. No.	Value of D.	Sum of Errors in units of one-tenth of second of arc.	No. of observations.
24	303°·2	−15	2
25	315·8		
26	328·4		
27	341·0		
28	353·7		
29	6·3		
30	19·0		
31	31·6		
32	44·2		
33	56·8		
34	69·5		
35	82·1	− 11	3
36	94·7	+ 14	3
37	107·4	+ 82	3
38	120·0	− 14	3
39	132·6	+ 8	5
40	145·3	+ 33	3
41	157°·9	+ 124	5
42	170·5	− 10	4
43	183·2	+ 43	4
44	195·8	+ 46	2
45	208·4	+ 43	4
46	221·0	+ 29	2
47	233·7	+ 74	4
48	246·3	− 20	2
49	259·0	+ 32	3
50	271·6	+ 3	2
51	284·2	+ 57	4
52	296·8	+ 13	1
53	309·5		
54	322·1		
55	334·7		
56	347·4		

Since the mean error is $+6\cdot4$ (in units of $0''\cdot1$) all errors have been decreased by 6, so as to render this period comparable with other periods. Line 9 therefore reads six errors with a sum $+7$.

It will be noted that the value of D is proportional to the reference number.

The periods are now arranged in six groups of eight periods each. The following table gives the results for each group and the sums for the whole forty-eight periods, or 54·5 years. The last column but one gives the mean error corresponding to the separate values of D, and the last column gives the same quantities corrected for a change of $0''\cdot45$ in semi-diameter, which appears to be indicated by the three preceding columns. Two lines nearest to full Moon are not filled in, as the observations do not always correspond to the same limb.

Table III. that follows gives the values of sin D, cos D, sin 2D, cos 2D, cos 3D, cos 4D for the different values of D: these quantities are required for a least-squares solution. Also in the same table are given the products of some of the above quantities by "*n*," the number of observations in the group, and E, the sum of the errors of the group.

The sums of lines 0–14, 15–28, 29–42, 43–56 are also taken for certain columns under the titles Σ_1, Σ_2, Σ_3, and Σ_4.

In the last two lines Σ denotes $\Sigma_1+\Sigma_2+\Sigma_3+\Sigma_4$, and Σ' denotes $\Sigma_1-\Sigma_2+\Sigma_3-\Sigma_4$.

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TABLE

Periods.	86—93.		94—101.		102—109.		110—117.	
	Sum of Errors.	No. of Obs.	Sum of Errors.	No. of Obs.	Sum of Errors.	No. of Obs.	Sum of Errors.	No. of Obs.
0	...	...	...	...	...	...	...	...
1	...	...	...	...	...	...	...	...
2	...	...	...	...	...	...	...	...
3	− 9	1	+ 39	1	...	...	...	...
4	− 59	8	− 54	9	+ 50	3	− 45	4
5	−179	14	−130	8	+ 72	10	+ 50	12
6	− 70	21	−338	15	− 79	17	−184	20
7	− 43	26	−140	25	− 18	19	−142	24
8	− 92	24	−184	31	−103	32	−176	32
9	− 23	31	−110	29	+ 59	28	−243	32
10	−188	30	−170	34	−121	31	+ 28	32
11	−136	25	−213	33	−166	32	+ 36	30
12	−320	30	−305	33	− 74	32	+ 32	28
13	− 6	29	−249	33	− 60	27	+109	34
14	−128	26	−162	30	− 88	36	+ 98	27
15	+238	25	+219	32	− 84	29	+158	30
16	+262	30	+240	33	+119	38	+ 86	24
17	+371	23	+424	31	+ 10	22	+221	26
18	+176	19	+275	21	+100	22	− 51	18
19	+ 38	24	+250	20	+234	28	+ 39	19
20	+ 93	26	+213	17	+ 39	22	− 61	12
21	+125	25	+ 44	23	−135	15	+ 22	13
22	+158	22	+168	27	− 41	16	+ 8	14
23	− 49	18	+243	19	+137	9	+144	15
24	+ 24	10	+ 47	8	+ 17	6	− 66	7
25	...	...	+ 9	1	− 11	1	...	...
26	...	...	...	...	...	...	...	...
27	...	...	...	...	...	...	...	...
28	...	...	...	...	...	...	...	...
29	...	...	...	...	...	...	...	...
30	...	...	...	...	...	...	...	...
31	...	...	+ 2	1	...	...	...	...
32	− 53	4	+ 14	1	+ 19	1	...	...
33	−104	7	− 34	6	+ 55	5	− 49	10
34	−117	21	−254	19	+ 66	13	− 69	12
35	−303	17	−214	24	+ 43	25	−139	18

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II.

Sum of Errors (118–125)	No. of Obs. (118–125)	Sum of Errors (126–133)	No. of Obs. (126–133)	Sum of Errors (86–133)	No. of Obs. (86–133)	Mean in units of 0''·01.	Value of ±0''·346 +0''·120 sin D −0''·030 sin 2D in units of 0''·01.
...	...	...	...	...	...	...	...
...	...	...	...	...	...	...	...
...	...	...	...	...	...	...	...
...	...	...	...	+ 30	2	...	...
− 2	5	− 67	4	− 177	33	− 54	+41
− 11	8	−207	19	− 405	71	− 57	+43
− 59	23	− 73	18	− 803	114	− 70	+45
+100	22	−139	17	− 382	133	− 27	+46
− 68	28	− 27	29	− 650	176	− 37	+48
+ 60	23	−214	25	− 471	168	− 28	+48
+ 24	17	−105	26	− 532	170	− 31	+47
− 22	29	−199	27	− 700	176	− 40	+46
+ 36	27	−269	29	− 900	179	− 50	+43
− 11	18	−140	32	− 357	173	− 21	+39
− 66	30	−120	36	− 466	185	− 25	+36
+157	28	+303	31	+ 991	175	+ 57	−38
+158	25	+291	33	+1156	183	+ 63	−41
+ 91	20	+252	29	+1369	151	+ 91	−44
+130	22	+ 74	27	+ 704	129	+ 55	−47
+ 16	20	+158	15	+ 735	126	+ 58	−48
− 85	14	+186	15	+ 385	106	+ 36	−48
−150	17	+107	24	+ 13	117	+ 1	−47
+ 76	14	+ 40	13	+ 409	106	+ 39	−46
− 33	16	+ 96	12	+ 538	89	+ 60	−44
+ 39	4	+ 30	6	+ 91	41	+ 22	−42
− 18	1	+ 53	1	+ 33	4	...	...
...	...	...	...	...	...	...	...
...	...	...	...	...	...	...	...
...	...	...	...	...	...	...	...
...	...	...	...	...	...	...	...
...	...	...	...	...	...	...	...
...	...	...	...	+ 2	1	...	...
− 1	3	− 50	4	− 71	13	− 55	+40
−121	9	− 44	13	− 297	50	− 59	+42
− 58	18	−121	17	− 553	100	− 55	+44
−177	15	−144	19	− 934	118	− 79	+46

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Periods.	Sum of Errors.	No. of Obs.	Sum of Errors.	No. of Obs.	Sum of Errors.	No. of Obs.	Sum of Errors.	No. of Obs.
	86—93.		94—101.		102—109.		110—117.	
36	−276	23	− 96	24	+ 45	22	−112	20
37	+ 39	22	−240	27	+112	35	− 76	25
38	−430	25	−242	25	−235	38	− 4	26
39	− 83	29	−273	28	−105	28	−121	25
40	−175	27	−328	34	− 60	36	− 11	33
41	− 75	36	−206	30	−156	31	0	21
42	−102	35	−263	27	− 1	35	+104	35
43	+250	35	+ 62	31	+ 20	27	− 91	33
44	+224	37	+323	32	+ 34	27	+129	23
45	+243	28	+143	19	+111	24	+177	28
46	+330	28	+129	31	+236	20	+183	19
47	+196	25	+286	21	− 30	15	+ 6	17
48	+108	26	+178	32	− 71	26	+ 75	21
49	+ 21	28	+212	16	− 47	21	+ 89	15
50	− 77	20	+267	21	+ 83	17	+ 20	17
51	+ 53	16	+198	18	+ 75	20	+ 10	11
52	+111	7	+120	10	+ 25	9	− 67	10
53	+ 38	5	+197	9	...	...	− 4	4
54	+ 28	1	− 13	1	...	...	...	...
55	...	...	...	...	...	...	...	...
56	...	...	...	...	...	...	...	...
Total	+ 29	989	+ 84	1000	+ 76	950	+113	876

TABLE

n.	Ref. No	100 sin D.	*n* sin D.	100 cos D.	*n* cos D.	100 sin 2D.	*n* sin 2D.	100 cos 2D.	*n* cos 2D
	0	0		+100		0		+100	
	1	+22		98		+43		90	
	2	43		90		77		64	
2	3	61	+ 1	79	+ 2	97	+ 2	+ 25	+ 1
33	4	77	25	64	21	98	32	− 19	− 6
71	5	89	63	45	32	81	58	59	42
114	6	97	111	25	29	48	55	88	100
133	7	100	133	+ 3	+ 4	+ 6	+ 8	100	133
176	8	98	172	− 19	−33	−38	−67	93	164
168	9	92	155	40	67	74	124	68	114
170	10	81	138	59	100	95	162	− 30	−51

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Sum of Errors.	No. of Obs.	Sum of Errors.	No. of Obs.	Sum of Errors.	No. of Obs.	Mean in units of 0″·01.	Value of ±0″·346 +0″·120 sin D −0″·030 sin 2D in units of 0″·01.
118–125.		126–133.		86–133			
+ 55	21	−182	25	− 566	135	− 42	+47
+101	27	−207	31	− 271	167	− 16	+48
−130	30	−215	32	−1256	176	− 71	+48
− 47	28	−189	31	− 818	169	− 48	+47
−251	28	−296	38	−1121	196	− 57	+44
+ 16	28	− 29	26	− 450	172	− 26	+41
+187	28	−125	40	− 200	200	− 10	+38
+ 25	30	− 47	31	+ 219	187	+ 12	−36
− 47	25	+278	37	+ 941	181	+ 52	−39
− 38	21	+235	27	+ 871	147	+ 59	−43
− 68	25	+214	32	+1024	155	+ 66	−46
+ 43	17	+232	29	+ 733	124	+ 59	−47
− 44	19	+139	19	+ 385	143	+ 27	−48
+ 9	12	− 11	18	+ 273	110	+ 25	−48
− 4	16	+158	23	+ 447	114	+ 39	−46
+ 42	12	+203	27	+ 581	104	+ 56	−45
− 14	5	+ 82	11	+ 257	52	+ 42	−43
+ 18	2	+ 29	4	+ 278	24	+116	−41
...	...	...	...	+ 15	2	...	...
...	...	...	...	...	...	...	...
...	...	...	...	...	...	...	...
−142	830	− 60	1002	+ 100	5647	...	...

III.

100 cos 3D.	n cos 3D.	100 cos 4D.	n cos 4D.	E.	E sin D.	E sin 2D.
+100		+100		...	...	...
79		+ 64		...	...	...
+ 25		− 19		...	...	...
− 40	− 1	88	− 2	+ 30	+ 18	+ 29
88	29	93	31	− 177	− 136	− 173
99	70	− 30	− 21	− 405	360	328
68	78	+ 55	+ 63	− 803	779	385
− 8	−11	99	132	− 382	382	− 23
+ 55	+97	+ 72	+127	− 650	637	+ 247
95	160	− 8	− 13	− 471	433	349
95	162	82	139	− 532	431	505

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n.	Ref. No.	100 sin D.	n sin D.	100 cos D.	n cos D.	100 sin 2D.	n sin 2D.	100 cos 2D.	n cos 2D.
176	11	66	116	75	132	99	174	+ 14	+25
179	12	48	86	88	158	84	150	55	98
173	13	27	47	96	166	52	90	85	147
185	14	+ 6	+11	100	−185	−11	−20	99	+183
175	15	−16	−28	99	−173	+32	+56	95	+166
183	16	38	70	93	170	70	128	72	132
151	17	57	86	82	124	94	142	+ 35	+ 53
129	18	74	95	68	88	100	129	− 8	− 10
126	19	87	110	50	63	87	110	50	63
106	20	95	101	30	32	57	60	82	87
117	21	100	117	− 8	− 9	+16	+19	99	116
106	22	99	105	+ 14	+15	−27	−29	96	102
89	23	94	84	35	31	66	59	75	67
41	24	84	34	55	23	92	38	− 40	− 16
4	25	70	− 3	72	+ 3	100	− 4	+ 3	0
	26	52		85		89		45	
	27	32		95		61		79	
	28	−11		99		−22		98	
	29	+11		99		+22		98	
	30	32		95		61		79	
1	31	52	+ 1	85	+ 1	89	+ 1	45	0
13	32	70	9	72	9	100	13	+ 3	0
50	33	84	42	55	28	92	46	− 40	− 20
100	34	94	94	35	35	66	66	75	75
118	35	99	117	+ 14	+17	+27	+32	96	113
135	36	100	135	− 8	−11	−16	−22	99	134
167	37	95	159	30	50	57	95	82	137
176	38	87	153	50	88	87	153	50	88
169	39	74	125	68	115	100	169	− 8	− 14
196	40	57	112	82	161	94	184	+ 35	+ 69
172	41	38	65	93	160	70	120	72	124
200	42	+16	+32	99	−198	−32	−64	95	+190
187	43	− 6	−11	100	−187	+11	+21	99	+185
181	44	27	49	96	174	52	94	85	154
147	45	48	71	88	129	84	123	55	81
155	46	66	102	75	116	99	153	+ 14	+ 22
124	47	81	100	59	73	95	118	− 30	− 37
143	48	92	132	40	57	74	106	68	97
110	49	98	108	− 19	−21	+38	+42	93	102

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100 cos 3D.	n cos 3D.	100 cos 4D.	n cos 4D.	E.	E sin D.	E sin 2 D.
+ 55	+ 97	96	169	− 700	462	693
− 8	− 14	− 40	− 72	− 900	432	756
68	118	+ 45	+ 78	− 357	96	186
99	− 183	98	+ 181	− 466	− 28	+ 51
88	− 154	79	+ 138	+ 991	− 159	+ 317
− 40	− 73	+ 3	+ 5	+ 1156	439	809
+ 25	+ 38	− 75	− 113	+ 1369	780	1287
79	102	99	128	+ 704	521	704
100	126	− 50	− 63	+ 735	639	639
79	84	+ 35	+ 37	+ 385	366	219
+ 25	+ 29	95	111	+ 13	13	+ 2
− 40	− 42	85	90	+ 409	405	− 110
88	78	+ 14	+ 12	+ 538	506	355
99	41	− 68	− 28	+ 91	76	84
68	− 3	100	− 4	+ 33	− 23	− 33
− 8		− 59		...	...	...
+ 55		+ 25		...	...	...
95		90		...	...	...
95		90		...	...	...
+ 55		+ 25		...	...	...
− 8	0	− 59	− 1	+ 2	+ 1	+ 2
68	− 9	100	13	− 71	− 50	− 71
99	50	− 68	− 34	− 297	249	273
88	88	+ 14	+ 14	− 553	520	365
− 40	− 47	85	100	− 934	925	− 252
+ 25	+ 34	95	128	− 566	566	+ 91
79	132	+ 35	+ 58	− 271	257	154
100	176	− 50	− 88	− 1256	1093	1093
79	134	99	167	− 818	605	818
+ 25	+ 49	− 75	− 147	− 1121	639	1054
− 40	− 69	+ 3	+ 5	− 450	171	315
88	− 176	79	+ 158	− 200	− 32	+ 64
99	− 185	98	+ 183	+ 219	− 13	+ 24
68	123	+ 45	+ 81	+ 941	254	489
− 8	− 12	− 40	− 59	+ 871	418	732
+ 55	+ 85	96	149	+ 1024	676	1014
95	118	82	102	+ 733	594	696
95	136	− 8	− 11	+ 385	354	285
+ 55	+ 60	+ 72	+ 79	+ 273	268	+ 104

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n.	Ref. No.	100 sin D.	n sin D.	100 cos D.	n cos D.	100 sin 2D.	n sin 2D.	100 cos 2D.	n cos 2D.
114	50	100	114	+ 3	+ 3	− 6	− 7	100	114
104	51	97	101	25	26	48	50	88	92
52	52	89	46	45	23	81	42	59	31
24	53	77	18	64	15	98	24	− 19	− 5
2	54	61	− 1	79	+ 2	97	− 2	+ 25	0
	55	43		90		77		+ 64	
	56	−22		+ 98		−43		+ 90	
Σ_1 1580	...		+ 1058		− 753		− 632		−156
Σ_2 1227	...		− 833		− 587		+ 514		−110
Σ_3 1497	...		+ 1044		− 693		− 649		−198
Σ_4 1343	...		− 853		− 688		+ 532		− 36
	...		...		...		...		...
Σ + 5647	...		+ 416		−2721		− 235		−500
Σ' + 507	...		+ 3788		...		−2327		...

The hypothesis is now made that E, the sum of the errors in any group, may be equated to

$$n\{\epsilon \pm \mu + \delta_1 \sin D + \delta_2 \sin 2D\}$$

where

ϵ represents the mean error ;
μ, the error of semi-diameter, which is here supposed constant ;
δ_1, a correction to the parallactic coefficient ;
δ_2, a correction to the variation coefficient.

Let the observations now be divided into four groups 0–14, 15–28, 29–42, 43–56, and let Σ_1, Σ_2, Σ_3, Σ_4 denote summations over these four groups ; then the normal equations are :

$$5647\epsilon + 507\mu + (\Sigma_1 + \Sigma_2 + \Sigma_3 + \Sigma_4)(\delta_1 \sin D + \delta_2 \sin 2D) = +100$$
$$507\epsilon + 5647\mu + (\Sigma_1 - \Sigma_2 + \Sigma_3 - \Sigma_4)(\delta_1 \sin D + \delta_2 \sin 2D) = -24796$$
$$\epsilon(\Sigma_1 + \Sigma_2 + \Sigma_3 + \Sigma_4) \sin D + \mu(\Sigma_1 - \Sigma_2 + \Sigma_3 - \Sigma_4) \sin D + \delta_1 \Sigma \sin^2 D$$
$$+ \delta_2 \Sigma \sin D \sin 2D = -17231$$

and

$$\epsilon(\Sigma_1 + \Sigma_2 + \Sigma_3 + \Sigma_4) \sin 2D + \mu(\Sigma_1 - \Sigma_2 + \Sigma_3 - \Sigma_4) \sin 2D$$
$$+ \delta_1 \Sigma \sin D \sin 2D + \delta_2 \Sigma \sin^2 2D = +10475$$

The calculations of Table III., remembering that

$$\sin^2 D = \tfrac{1}{2}(1 - \cos 2D)$$
$$\sin^2 2D = \tfrac{1}{2}(1 - \cos 4D)$$
$$\sin D \sin 2D = \tfrac{1}{2}(\cos D - \cos 3D)$$

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100 cos 3D.	n cos 3D.	100 cos 4D.	n cos 4D.	E.	E sin D.	E sin 2D.
− 8	− 9	99	113	+ 447	447	− 27
68	71	+ 55	+ 57	+ 581	564	279
99	51	− 30	− 16	+ 257	229	208
88	21	93	22	+ 278	214	272
− 40	− 1	88	− 2	+ 15	− 9	− 15
+ 25		− 19		...	...	...
+ 79		+ 64		...	...	...
+ 12		+ 134		− 5813	− 4158	+ 1907
− 12		+ 57		+ 6424	− 3927	+ 3395
+ 86		+ 13		− 6535	− 5106	+ 2630
− 74		+ 152		+ 6024	− 4040	+ 2543
...		...		...	...	...
+ 12		+ 356		+ 100	− 17231	+ 10475
...		...		− 24796	...	...

enable us to write the normal equations in the numerical form

$$\begin{aligned}
5647\epsilon + 507\mu + 416\delta_1 - 235\delta_2 &= +100 \\
507\epsilon + 5647\mu + 3788\delta_1 - 2327\delta_2 &= -24796 \\
416\epsilon + 3788\mu + 3074\delta_1 - 1366\delta_2 &= -17231 \\
-235\epsilon - 2327\mu - 1366\delta_1 + 2645\delta_2 &= +10475
\end{aligned}$$

We do not want the value of ϵ, and it is clear that we may drop the first equation and the terms containing ϵ in the others.

Solving the three equations that remain, I obtain

$$\begin{aligned}
-\mu &= +0''{\cdot}346 \\
+\delta_1 &= -0''{\cdot}120 \\
+\delta_2 &= +0''{\cdot}030
\end{aligned}$$

or the tabular semi-diameter requires a correction

$$-0''{\cdot}346$$

The tabular parallactic coefficient requires a correction

$$+1''{\cdot}562 + 0''{\cdot}120$$

(of which the first term had been applied to the individual observations before grouping).

The tabular variation coefficient requires a correction

$$-0''{\cdot}030$$

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About this method of treatment the following points are to be remarked:

(1) The observations of line 14 are not all first-limb observations, nor those of line 43 all second-limb. It has not been thought worth while to undertake the labour of applying the necessary correction. Probably not more than fifty observations, or less than 1 per cent., have been assigned to the wrong limb.

(2) The hypothesis that the error of semi-diameter is a constant is the best that I can make. It will be noticed that the observations grouped in line 7, for example, contain some daylight and some night observations, according to the season of the year. I am not concerned with any difference in the correction applicable to the individual observations of this group, but merely to the mean correction. The mean correction for different ages of the Moon is no doubt a function of the age of the Moon and not a constant. If there were any means of obtaining its numerical values they could be analysed into the form $f(\mathrm{D})+\alpha \sin \mathrm{D}+\beta \sin 2\mathrm{D}$, when α, β would be corrections applicable to the results of this paper. As far as I can see the difficulty is inherent in this method of finding the solar parallax, which is rendered uncertain by an amount $\frac{1}{14}\alpha$. I do not think that α can possibly be as large as 0''·30, and it is possibly a good deal smaller, so that the parallax deduced in this paper should not be as much as 0''·02 in error.

The weights of the values found for μ . δ_1 and δ_2 are 781, 515,[*] and 1614 times the weight of a single observation.

The pure error of observation has been found from the transit circle and altazimuth to have a probable amount 0''·75. But we have to deal here with the probable discordance from the tables as corrected for the terms set out at the beginning of this paper. Perhaps 1''·5 is a fair estimate. If so, the probable error of this determination of the parallactic coefficient is

$$\pm 0''{\cdot}07$$

and of the resulting solar parallax ±0''·005.

Having now completed the investigation of the correction required by Hansen's parallactic terms, it remains to deduce the value of the solar parallax.

Newcomb (*Astronomical Constants*, p. 190) gives the following equation:

$$\text{Parallactic Coefficient} = -124''{\cdot}66\ \frac{\text{Solar Parallax}}{8''{\cdot}790}$$

This equation assumes the mass of the Moon to be $\frac{1}{81{\cdot}15}$.

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[*] Implying that owing to the difficulty of separating the error of parallactic coefficient from the error of semi-diameter 5647 observations are worth as much as 1030 observations would be worth if the difficulty did not exist.

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times that of the Earth; and as Newcomb considers the denominator of this fraction uncertain by only ±0·15 (p. 189), this introduces an inappreciable uncertainty into the value of the solar parallax.

In deducing this equation, however, Newcomb assumes that Hansen's theory is correct, whereas he has himself thrown doubt upon this very point. I have transformed Brown's results (*Memoirs R.A.S.* vol. liii. pp. 89 and 96), and I find that they confirm Delaunay's theory with Newcomb's estimate of the correction on account of terms of high order not calculated by Delaunay, and Hansen is therefore 0″·55 in error. I therefore obtain the above equation in the revised form:

$$\text{Parallactic Coefficient} = -125''{\cdot}2 \; \frac{\text{Solar Parallax}}{8''{\cdot}790}$$

Hansen's tables and Hansen's theory differ by 0″·033 in the coefficient of the parallactic term for the same value of the solar parallax; and Hansen's theory leads to a coefficient of −122″·032 (Newcomb, *Astron. Papers Amer. Eph.* vol. i.), and Hansen's tables consequently to

$$-122''{\cdot}065 \times 1{\cdot}03573 = -126''{\cdot}43$$

The coefficient with which the observations have been compared is therefore

$$-126''{\cdot}43 + 1''{\cdot}56 = -124''{\cdot}87$$

and the further correction deduced in this paper alters this to

$$-124''{\cdot}87 + 0''{\cdot}120 = -124''{\cdot}75$$

The solar parallax is therefore

$$8''{\cdot}76$$

The correction to the mean semi-diameter used since 1847 is −0″·346. The semi-diameter corresponding to a parallax of 57′ 2″·3 used since 1856 is 15′ 34″·10. This quantity should therefore be about 15′ 33″·75.

Lastly Hansen's theory, Delaunay and Hill agree as to the coefficient of the variation, but in his tables Hansen has added 0″·34 to this coefficient to correct for an assumed difference between the Moon's centre of figure and centre of gravity. This paper confirms the coefficient of Hansen's tables. This suggests that the evection should be analysed to see whether it supports Hansen's assumption, and also it appears advisable to transform Hansen's tables in the same way that Newcomb has already transformed his theory.

It will be also noticed that if the semi-diameter and variation are determined from other sources, and values differing by $\Delta\mu$ and $\Delta\delta_2$ from the results of this paper are found for them, then

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if $\Delta\delta_1$ and $\Delta\pi$ be the corrections to the parallactic coefficient and solar parallax

$$\Delta\pi = +\frac{1}{14 \cdot 2}\Delta\delta_1 = \frac{1}{14 \cdot 2}\left\{-\frac{3788}{3074}\Delta\mu + \frac{1366}{3074}\Delta\delta_2\right\}$$

$$= -\frac{1}{11 \cdot 5}\Delta\mu + \frac{1}{32 \cdot 0}\Delta\delta_2$$

It should be mentioned that Newcomb in 1876 attributed the apparent increase of the variation to systematic errors of observation varying with the age of the Moon.

On Oscillating Satellites. (Second Paper.)

By H. C. Plummer, M.A.

1. In the earlier paper (*Monthly Notices*, vol. lxiii. p. 436), to which the present is a continuation, periodic orbits which are possible in the neighbourhood of centres of libration have been investigated with a degree of approximation corresponding to the second order of the relative coordinates. The results appear to suggest that a still closer approximation may be made with some advantage, although the discussion relates only to a small part of the restricted problem of three bodies, and has apparently no direct practical bearing. The notation of the former paper will be followed generally, but some changes will be admitted. The three collinear centres of libration alone will be considered here.

2. The general equations of motion of the satellite are taken in the form

$$\ddot{x} - 2\dot{y} = \Omega_{10}, \qquad \ddot{y} + 2\dot{x} = \Omega_{01}$$

where

$$2\Omega = \mu\left(r_1^2 + \frac{2}{r_1}\right) + \nu\left(r_2^2 + \frac{2}{r_2}\right), \quad \Omega_{rs} = \frac{\partial^{r+s}\Omega}{\partial x^r \partial y^s}.$$

Now at the collinear centres of libration $y = 0$ and $\Omega_{10} = \Omega_{01} = 0$; and $\Omega_{rs} = 0$ when s is odd. Hence in relative coordinates the equations of motion are

$$\left.\begin{aligned} \xi - 2\eta &= \Omega_{20}\xi + \tfrac{1}{2}(\Omega_{30}\xi^2 + \Omega_{12}\eta^2) + \tfrac{1}{6}(\Omega_{40}\xi^3 + 3\Omega_{22}\xi\eta^2) + \cdots \\ \ddot{\eta} + 2\dot{\xi} &= \Omega_{02}\eta + \Omega_{12}\xi\eta \qquad\qquad + \tfrac{1}{6}(3\Omega_{22}\xi^2\eta + \Omega_{04}\eta^3) + \cdots \end{aligned}\right\} \ldots \quad (1)$$

so long as the satellite remains within the region over which the expansions are possible. It is known that periodic solutions of these equations exist, and these can be appropriately expressed by Fourier series of the form

$$\left.\begin{aligned} \xi &= a_0 + a_1 \cos mt + a_2 \cos 2mt + a_3 \cos 3mt + \cdots \\ \eta &= \qquad b_1 \sin mt + b_2 \sin 2mt + b_3 \sin 3mt + \cdots \end{aligned}\right\} \ldots \quad (2)$$

the orbits being symmetrical with respect to the ξ-axis.